ZFITTER 2012

– dedicated to Pena Christova on the occasion of her 70th birthday –

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We recapitulate the ZFITTER project in past, present, future, seen in 2012.

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[*] Speaker.

1. Introduction

When the talk was given at “Loops and Legs” in April 2012, its title was “ZFITTER - 20 years after”. By transforming the talk into this contribution to the conference proceedings, we realized that the title is a bit misleading and modified it into “ZFITTER 2012”.

To fix a date of begin of ZFITTER is not unique. We chose the year 1985, when the article “Hunting the hidden standard Higgs” was published [1]. With this study, we began to take into account a finite, non-zero top quark mass m_t in the radiative corrections. To our knowledge, the paper contains the first plot confronting two LEP observables – weak mixing angle $\sin^2\theta_W$ and Z boson mass M_Z – with their dependence on the unknown top quark mass m_t and the also unknown Higgs boson mass M_H in the Standard Model [2 – 5]. We reproduce the plot here in figure 1.2, left[1]. Both top quark and Higgs boson were not yet discovered at that time, and the actual experimental values for M_Z and $\sin^2\theta_W$ had too huge errors to be included into the plot [6]: $M_Z = 92.9 \pm 16$ GeV and $\sin^2\theta_W = 0.23 \pm 0.015$. The numbers in the figure are based on the one-loop Standard Model prediction for Δr, the weak correction to G_μ, deserving few lines of Fortran code. We remark as a curiosity that from 1985 to 2011, the article was quoted only once (by authors outside our group).

The LEP collaborations made exciting measurements of the Z boson resonance and of its mass and width, with an unexpected final accuracy, $M_Z = 91.1876 \pm 0.0021$ GeV (corresponding to $\Delta M_Z/M_Z \approx 10^{-5}$) and $\Gamma_Z = 2.4952 \pm 0.0023$ GeV [8]. Figure 1.3 shows the rise of accuracy for M_Z due to LEP. Since the begin of the nineteen-nineties, a true scientific standard is the so-called blue-band plot of the LEPEWWG, based on ZFITTER [9 – 11] and another Standard Model package TOPAZ0 [12 – 14]. The March 2012 version is reproduced in figure 1.2, right. Both

Figure 1.1: *ZFITTER authors: Lida Kalinovskaya, Pena Christova, Dima Bardin, Tord Riemann, Sabine Riemann, Andrej Arbuzov. Photograph made on occasion of 5th Helmholtz International Summer School "Calculations for Modern and Future Colliders", July 23 - August 2, 2012, Dubna, Russia, CALC 2012. Right: Sasha Olshevsky, Arif Akhundov, Mark Jack. Fotos:* © *2012 tordriemann@googlemail.com; JINR, Dubna; A. Akhundov (priv.); Physics at FAMU.*

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[1] Figure 1.2 reprinted from Physics Letters, A. Akhundov, D. Bardin, and T. Riemann, “Hunting the hidden standard Higgs”, volume B166, p. 111, Copyright (1986), with permission from Elsevier.

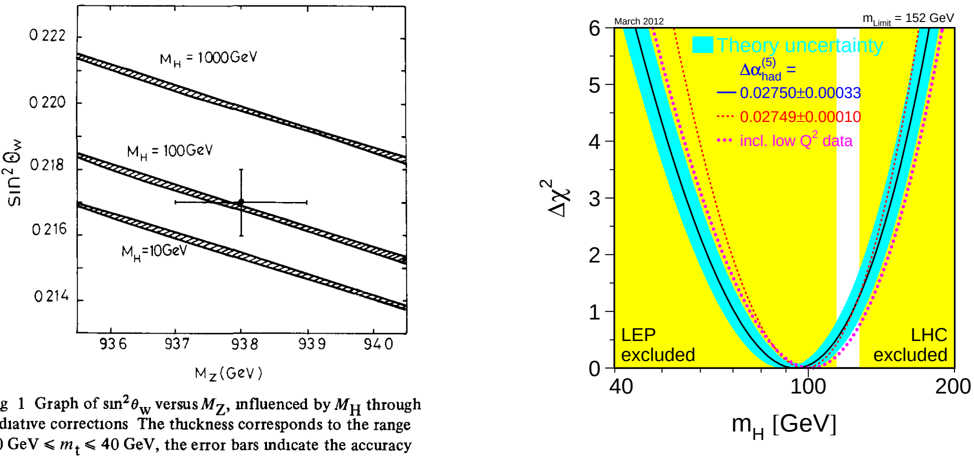

Fig 1 Graph of $\sin^2\theta_W$ versus M_Z, influenced by M_H through radiative corrections The thickness corresponds to the range 30 GeV ≤ m_t ≤ 40 GeV, the error bars indicate the accuracy expected at Z boson factories.

Figure 1.2: *Left: The first ever plotted LEP observables' dependence on the Higgs mass in the Standard Model (reproduced from [1]. Right: Blue-band plot of the LEPEWWG [7] with a Standard Model Higgs boson mass prediction based on combined world data from precision electroweak measurements.*

ZFITTER and TOPAZ0 are huge software packages with tens of thousands lines of Fortran code aiming at covering the complete known radiative corrections to the Z resonance peak in the reaction $e^+e^- \to \bar{f}f$. The top quark was predicted by M. Kobayashi and T. Maskawa in 1973 [15] and discovered in 1995 with a mass of about 173 GeV [16, 17]. Top quark mass data from precision electroweak measurements and from direct searches are collected in figure 1.3, right. Over the years the predictive power of the indirect searches for the Higgs boson mass improved considerably, and the discovery of the top quark was a crucial improvement for this. This is described in figure 1.4. In 2012, the LHC collaborations reported the discovery of a scalar particle with a mass of about 125 GeV [18, 19], which fits into these expectations from the indirect searches. It might well be that it is the particle predicted by Peter Higgs in 1964 [20, 21].

We live with the ZFITTER project for more than 20 years now, and ZFITTER is yet in use for a diverse variety of applications, ranging from the global analyses of the LEPEWWG to many graduation papers like e.g. [22]. Twenty years are a long term. It takes similarly long to prepare final results of big experiments at accelerators as LEP 1, LEP 2, HERA. As an example, we mention the final analysis of the LEP 1 data for two-fermion production in 2005 [23] by the LEP collaborations and the LEPEWWG, using ZFITTER v.6.42. The corresponding enterprise for LEP 2 data is yet being finalized, using ZFITTER v.6.43.

The big laboratories invented scientific programs for a dedicated long-term preservation of the experimental data, under the label "ICFA Study Group on Data Preservation and Long Term Analysis in High Energy Physics" [24]. One might assume that this is a self-evident issue of any physics collaboration. Physics is the science of reproducible observations in Nature and of their explanations/descriptions, and reproducibility deserves storage. But long-term storage is an unsolved problem, worth of any (reasonable) effort. DESY, as an example, founded in 2009 a "DESY Data Preservation Project", mainly focusing on the HERA experiments H1, Hermes, ZEUS

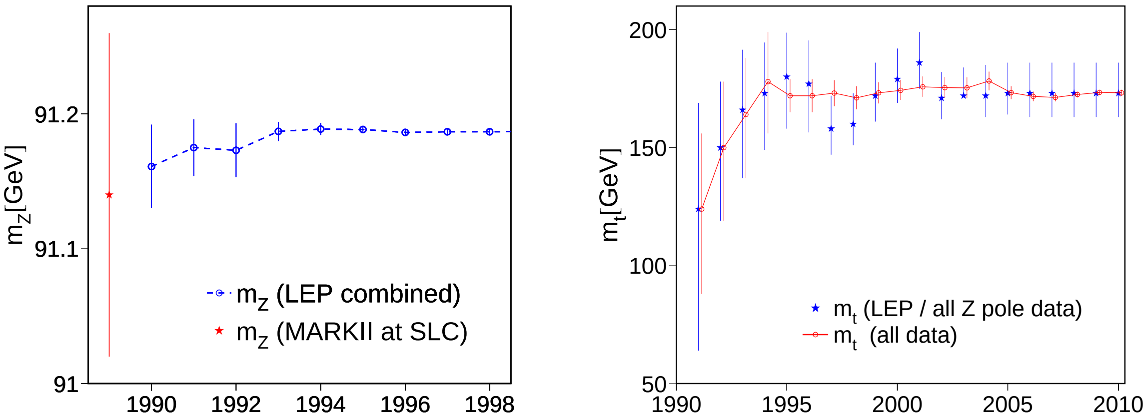

Figure 1.3: *Left: Z boson mass measurements at LEP. Earlier measurements are from UA1, UA2 at SPS (CERN) (see text, not shown in plot) and from MARKII at SLC (SLAC). Right: Top quark mass measurements. After discovery of the top quark, the LEP data were no more competitive. The agreement of direct measurements (in 'all data') and indirect measurements (in 'all Z pole data') supports the validity of the Standard Model at the quantum loop level.*

[25]. If such effort is justified for data, then it is also needed for the analysis tools, which were used for an extraction of the Model with its few parameters from the raw, or not-so-raw, Data. To our knowledge, the Big Labs do not plan to support long-term maintenance of software like ZFITTER. We, as authors, theoreticians and phenomenologists, have to mind by ourselves about maintenance of theory/phenomenology software.

Everybody knows that the very details of a data analysis cannot be described by few words. But for precision studies they are truly essential. Sometimes we say: "The description of the program is the program itself." This is a helpful statement if "the program itself" is preserved over a long term in its state of use. ZFITTER did and does a lot to fulfil such a demand. See the webpage `http://zfitter.com`.

Preservation demands effort. In the DESY Data Preservation Project there are 17 people involved. At the other hand, if a theoretician says: I care about the availability of my old software, people start to smile. This aim does not give true credit points for a scientific carrier, in what phase of the carrier ever.

In fact, not only the so-called main author of ZFITTER, Dima Bardin, our "primus inter pares", tends to lose interest in active support of ZFITTER over the decades. This applies to all of us, mainly because of our interest in studying or inventing something new. Nevertheless, we collected in 2005 some volunteers into a ZFITTER support group, which submitted in that year ZFITTER v.6.42 and in 2008 ZFITTER v.6.43 [10, 11].

Encouraged by the decreasing visibility of our ZFITTER support, in 2006 some experimentalists tried to re-program in C++ in a year's time the Standard Model library of ZFITTER from the published literature. Not just for fun, but in order to do better than ZFITTER: use a more modern programming language than Fortran, with more modularity than ZFITTER, a bit updated, with a GUI. In order to retain ZFITTER for a longer term. The project was proprietary until 2012, and it faced two major problems. It proved to be impossible to do so without using the ZFITTER software itself to a large extent. Further, without cooperation with ZFITTER authors and with the community of theoreticians, including extensive numerical cross-checks, such a project cannot succeed.[2]

Finally, there is much influence by institutes' directors and by the editors and publishers of physics journals on the engagement of scientists in the development of software. Not all of them seem to mind about proper acknowledgment and quotation of software. Some even say that software has no genuine scientific value by itself and advocate an absolutely free use of any software as common habit. If this would *become* common habit, nobody with inspiration and ambition would invest time to write complicated software for the use by other people, like the ZFITTER group - and other groups as well - does. We live in an academic world and we are valued by our scientific results, their originality, importance, curiosity, usefulness etc. Financing of our projects, of our working positions, our academic prestige depend on all that. We need proper quotation of our scientific results in case they are used. And we can only appeal and hope that the community understands this as a justified expectation, also for software.

As a key feature of user-friendly support, we stored for many years all the relevant versions of ZFITTER at a webpage for anonymous download. We collected about three dozen versions, covering more than 20 years. There are collegues who take the freedom to use ZFITTER as if it were open-source software in the strictest meaning of the word. Despite the facts that academic research deserves strict, proper quotation, and that there are licence regulations (for ZFITTER this includes the CPC licence). In some countries there are even legal regulations.[3]

It is the aim of these notes to give an overview on the ZFITTER project. Maybe they can help to see theoretical software in particle physics as an intellectual enterprise like the other inventions of physics research - experimental set-ups, data, hypotheses, models, theories.

We would like to finish the introduction with two quotes.

Several times we all thought that the ZFITTER project is in its final phase of dying out. See for example the remark of Dima Bardin at the symposium “50 Years of Electroweak Physics: a symposium in honour of Professor Alberto Sirlin's 70th Birthday”, in the year 2000 [26]:

> *“We would like to see the end of the ZFITTER project in the year 2000 and, therefore, a very natural question arises: What's next?”*

In the same year, members of the ZFITTER group were granted the prestigeous “JINR Award in Theoretical Physics” of the Joint Institute for Nuclear Research, Dubna, Russia. For a document, see here: certificate. The referee was Academician Prof. L. B. Okun from ITEP Moscow; he finished his estimate with the statement:[4]

> *“Overall, the project "ZFITTER Fortran program" represents a unique theoretical tool of world class. The project formed the basis of a close cooperation of experimetalists and theoreticians (with a series of workshops at CERN). With the accumulation of experimental data, the accuracy of the programs has been increased. The project has always found*

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[2] For details see e.g. `http://zfitter.com`.

[3] Due to controversial positions, we closed the links for anonymous download from ZFITTER webpages in 2011, and in 2012 also the links from the Andrew file system at CERN.

[4] The original document is in Russian, see statement.

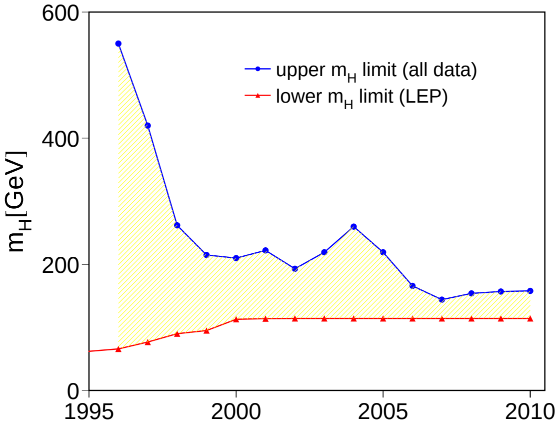

Figure 1.4: *Higgs boson mass measurements. The upper limits and the fit values for M_H derive from a combination of virtual corrections to LEP and similar data, top and W mass measurements, performed by the LEPEWWG. The lower mass limit is due to LEP direct searches. The lower limits from data combinations are not shown.*

great interest at conferences. Its importance and the interest to it shows with numerous references in articles, reviews and monographies.
In the long term, with the advent of more precise experiments, ZFITTER will allow to take into account all two-loop electroweak corrections.
The series of theoretical articles on precision tests of the Standard Model at electron-positron colliders certainly deserves the award of the JINR prize 2000.
Academician L.B. Okun"

Our figures illustrate the development of mass predictions for Z boson (figure 1.3, left), top quark (figure 1.3, right), Higgs boson (figure 1.4). Here, ZFITTER has been useful until now. Okun's proposition that ZFITTER will be used also in future is being fulfilled. We can only hope that our write-up might help to convince the present particle physics community that ZFITTER is worth some support by now and in future.

At the end of the introduction, we would like to reproduce the long(est) authors list of ZFITTER, see also `http://zfitter.com`:

A. Akhundov, A. Arbuzov, M. Awramik, D. Bardin, M. Bilenky, A. Chizhov, P. Christova, M. Czakon, O. Fedorenko (1951-1994), A. Freitas, M. Grünewald, M. Jack, L. Kalinovskaya, A. Olshevsky, S. Riemann, T. Riemann, M. Sachwitz, A. Sazonov, Yu. Sedykh, I. Sheer, L. Vertogradov, H. Vogt.

The list is not complete. According to the conventions of the software library of "Computer Physics Communications", we should also include here all the co-authors who helped to prepare the program descriptions in 1989, 1999, 2005 [9 – 11].

2. ZFITTER in a nutshell, or: Is there a ZFITTER approach?

We never used the label “ZFITTER approach”. The reason is simple: There is no ZFITTER approach. If any, there is a kind of Dubna approach, or of Bardin’s group’s approach.

Nevertheless, other people use this phrase. Let us collect some distinguishing moments which might be the origin of some popularity of ZFITTER, but also of one or the other of our scientific projects:

- Unitary gauge.
  We are working in the unitary gauge when studying the renormalization of the Standard Model. Most of the other groups use the ’t Hooft-Feynman gauge. But when looking at observable quantities, there is no difference left, due to the gauge invariance of perturbation theory. So, if everything is correct, there is no difference for the users.

- On-mass-shell renormalization scheme.
  We are applying the on-mass-shell renormalization scheme, with few modifications. Other groups do the same for electroweak corrections.

- Analytical treatment of QED corrections.
  ZFITTER is not a Monte-Carlo program. The Dubna group has an enormous experience in the analytical treatment of QED corrections, allowing us to come relatively close to the experimental set-ups by dedicated analytical integrations. Several different approaches may be chosen by users. The necessary computational time for fits to data is small compared to that of other projects.

- Realistic observables and pseudo-observables.
  There is a plethora of observables, of quite different polarized and non-polarized cross-section combinations and asymmetries. Both so-called realistic observables (including real corrections) and pseudo-observables (after unfolding the realistic observables) may be used. With the different interfaces one may optimize a study appropriately.

- Form factors. Modularity.
  We describe the effective Born cross-section in the Standard Model approach by (essentially) four (compex-valued) gauge invariant form factors per production channel. Plus a separated running QED coupling. This allows a modular programming, the efficient introduction of New Physics into the package or the convenient export of the Standard Model corrections into another approach to the real corrections.

- Higher-order corrections.
  Originally, we calculated the complete electroweak one-loop corrections to the Z resonance physics. By time, there became more and more electroweak, QCD, and mixed higher-order corrections available, and we had to implement them into ZFITTER. In the nineteen-nineties these implementations dominated our efforts for ZFITTER. It is not the genuin theoretical work we like, but has to be done.

- Interfaces. Modularity.
  ZFITTER is not a fitting program. But from the very beginning, we were aware of the fact that a data analysis at e.g. LEP may rest on different sets of assumptions, being incompatible to each other. The notion of interfaces was developed. The interfaces call the kernel of ZFITTER with different compositions of input variables, real corrections, an effective Born cross section. The users of ZFITTER can choose among few sample interfaces, or they write their own ones.

- Flags.
  The use of ZFITTER may be controlled by flags to be set by users. Although this implies problems for the update by the authors, for users this is truly convenient.

- Descriptions.
  ZFITTER is described for users at different levels of complexity. There are about 350 pages of instructions.

- Simplicity of file structure.
  ZFITTER is easy to use. It has a simple file structure, is self-contained, and has a sample output. The installation at a computer is done and controlled within minutes. The installation of the user software, which is calling ZFITTER and performing data fits, writing tables and drawing figures, might be much more involved.

- Numerical cross-checks.
  With very precise data available, as it was typical for LEP physics, a careful numerical control of the theory software became mandatory. Here, a lot of collegues, including competitors of ZFITTER, invested huge collective efforts. Without that, one could not trust the impressive physical results of that era, or the long-term reliability of the code.

- Source-open programming.
  The scientific seriosity of ZFITTER is trustable because its source code is publicly available. Because we expect that the usual academic conditions of use are respected, notably the CPC licence, we say it is source-open software. The meaning of the word open-source software is controversial and it should not be used for ZFITTER.

- Social aspects.
  A software package of some complexity, written for use by other people, must be supported and, in case, updated. The authors need some contact with the users. And, last but not least, some licence regulations have to be fixed if the authors want to get their academic credit, e.g. in form of proper citations. Since the authors of ZFITTER are employed at some institutions distributed over several countries, it is of vital importance that these institutions do not interfere in a destructive way. We are happy that this did not happen for a very long period, in view of several social restructurings of institutions and even countries.

ZFITTER is a Fortran library of Standard Model predictions for the scattering process

$$e^+e^- \to \bar{f}f \;\; (+\gamma, \;\; +n\,\gamma) \tag{2.1}$$

at energies in the range $\sqrt{s} \approx 20$ GeV to 150 GeV above quark bound states [meson factories] and below the top threshold. The package is to be called by interfaces

- in the Standard Model;
- in several model-independent approaches;
- with Z' bosons and similar physics extensions;
- etc.

One may evaluate

- realistic observables – polarized and non-polarized cross-sections and cross-section asymmetries with a variety of cuts on the final state
- (pseudo-)observables like $M_Z, \quad \Gamma_Z, \sigma^{\mathrm{tot}}_{\mathrm{had}}, \quad R_{\mathrm{had}}, \quad A^{\mathrm{lept}}_{\mathrm{FB}}, \quad \lambda_\tau, \quad \sin^2\theta^{\mathrm{eff}}_{\mathrm{ew}}, \quad \cdots$
- the form factors, for use in another analysis program.

with different choices of input variables, e.g.

- $M_Z, \quad G_\mu, \quad m_t, \quad M_H, \quad \alpha_{\mathrm{em}}, \quad \alpha_s, \quad \cdots$
- $M_Z, \quad M_W, \quad m_t, \quad M_H, \quad \alpha_{\mathrm{em}}, \quad \alpha_s, \quad \cdots$

3. Electroweak corrections

The first weak one-loop calculations were published as Dubna preprints by D. Bardin and his PhD student O. Fedorenko in 1978 [27 – 29]. Together with P. Christova, then also PhD student of D. Bardin, the by now famous articles on the complete on-mass-shell renormalization of the electroweak Standard Model were published in Nuclear Physics B [30, 31], for fermion scattering. See also [32]. The corresponding studies for weak boson production and fermion-boson scattering are unpublished [33, 34].

These calculations were complete, but assumed all fermions to be massless. When experiments showed that at least the top quark should be heavy, the top mass dependence was included [1, 35 – 37].

All this was done in the unitary gauge, while the other groups usually worked with the 't Hooft-Feynman gauge. Later, this difference was of some value because an agreement of two calculations performed in truly quite different gauges establishes a powerful cross-check of the numerics.

The Zeuthen partners, staying at Dubna from 1983 to 1987, worked out the renormalization of the electroweak Standard Model in the 't Hooft-Feynman gauge [38]. But because there was never a numerical program created, the results of this work were more or less useless; they had a mere educational aspect. Nevertheless, the experiences from that activity were used in order to perform the first calculation of flavor-changing Z boson decays into different lepton flavors.[5] This

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[5] For scanned preprints, see the webpage lfvz-intro.html. We mention for curiosity that the numerics of this one-loop project was performed with a pocket calculator TI-57 with 50 program steps. The program had to be typed in after switching on. The price of the device was 120 DM in the CERN shop.

was unpublished [39]; see also [40]. An application to flavor-violating Z decays into different quark flavors was finally published [41]. Later, when we were working on precision predictions for LEP, the results could be easily transformed into the calculation of virtual top mass corrections in (flavor-diagonal) $b\bar{b}$ production at LEP and in Z decay [35]. And yet later, they were a starting point for studies of lepton number violation in e^+e^- annihilation with heavy neutrinos [42] and with supersymmetry [43].[6]

3.1 Sirlin's approach to neutral current matrix elements

The notion of form factors ρ and κ in the weak neutral current were, to our knowledge, introduced by A. Sirlin:[7]

- ρ – contains the electroweak corrections to the Fermi constant G_μ
- κ – contains the electroweak corrections to the weak mixing angle $\sin^2\theta_W$

This approach allows to retain in the on-mass-shell renormalization scheme the Born definitions also in higher orders:

$$G_F^{\mathrm{eff}} = \rho_Z\, G_\mu, \tag{3.1}$$

$$\sin^2\theta_W^{\mathrm{eff}} = \kappa_Z\, \sin^2\theta_W, \tag{3.2}$$

where

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8M_W^2}, \tag{3.3}$$

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}. \tag{3.4}$$

3.2 The HECTOR and ZFITTER approach to effective Born cross sections – four weak neutral form factors

For general 4-fermion scattering amplitudes, one needs a more general description. This was first introduced, to our knowledge, by the Dubna/Zeuthen group, in 1988, in the article "Electroweak Radiative Corrections To Deep Inelastic Scattering At HERA. Neutral Current Scattering" by D. Bardin, C. Burdik (Dubna), P. Khristova (Shoumen), T. Riemann (Zeuthen) [45]. The software is retained until today as the Fortran package HECTOR [46]. So, strictly speaking, one might call this the HECTOR approach.

We use four complex form factors $\rho, \kappa_{\mathrm{ini}}, \kappa_{\mathrm{fin}}, \kappa_{\mathrm{ini-fin}}$ for the parameterization of the weak amplitude, including the WW and ZZ box diagrams. In the article "A Realistic Approach to the Standard Z Peak" by D. Bardin, M. Bilenky, G. Mitselmakher (Dubna), T. Riemann, M. Sachwitz (Zeuthen) [37], we excluded the weak WW and ZZ box diagrams from the form factors, making them independent of the scattering angle. This is of advantage at LEP where these box diagrams

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[6] Several of the results in supersymmetry found in the literature turned out to be just wrong when we had a look at them.

[7] For a historical perspective, see [44].

have minor numerical influence. When form factors are independent of the scattering angle, analytical phase space integrations become possible. In ZFITTER, there is an option to switch between the approaches.

The Born amplitude is factorized in two pieces with vector coupling v_i and axial vector coupling a_i of a fermion i to the Z-boson:

$$A\otimes B \equiv \left[\bar{u}_i\gamma_\mu(v_i+a_i\gamma_5)u_i\right] \times \left[\bar{u}_f\gamma^\mu(v_f+a_f\gamma_5)u_f\right], \tag{3.5}$$

and is generalized by loop corrections to

$$A_{vv}\gamma\otimes\gamma+A_{av}\gamma\gamma_5\otimes\gamma+A_{va}\gamma\otimes\gamma\gamma_5+A_{aa}\gamma\gamma_5\otimes\gamma\gamma_5, \tag{3.6}$$

or, equivalently,

$$B_{LL}\gamma(1+\gamma_5)\otimes\gamma(1+\gamma_5)+B_{\gamma L}\gamma\otimes\gamma(1+\gamma_5)+B_{L\gamma}\gamma(1+\gamma_5)\otimes\gamma+B_{\gamma\gamma}\gamma\otimes\gamma. \tag{3.7}$$

With Z boson and photon exchanges:

$$\mathscr{M} = \mathscr{M}_\gamma+\mathscr{M}_Z, \tag{3.8}$$

$$\mathscr{M}_\gamma \sim F_A\left[\gamma\otimes\gamma\right], \tag{3.9}$$

$$\mathscr{M}_Z \sim G_\mu\,\rho_Z\left[\gamma\gamma_5\otimes\gamma\gamma_5+v_q\gamma\otimes\gamma\gamma_5+v_l\gamma\gamma_5\otimes\gamma+v_{ql}\gamma\otimes\gamma\right]. \tag{3.10}$$

In Born approximation, it is

$$v_{ql}\approx v_q\times v_l. \tag{3.11}$$

The form factors $F_A,\ \rho,\ \kappa_q,\ \kappa_l,\ \kappa_{ql}$ are complex-valued functions of s and t:

$$F_A(s) = \frac{\alpha_{QED}(s)}{\alpha_{em}} \tag{3.12}$$

$$= 1+\delta\alpha_{QED}(s),$$

$$\alpha_{em} = \frac{1}{137\cdots}, \tag{3.13}$$

$$a_f \equiv 1, \qquad f=q,l \tag{3.14}$$

$$v_f(s,t)^{\mathrm{eff}} = 1-4\sin^2\theta_w|Q_f|\kappa_f(s',t), \quad f=q,l \tag{3.15}$$

$$v_{ql}(s,t)^{\mathrm{eff}} = v_q+v_l-1+16\sin^4\theta_W|Q_qQ_l|\kappa_{ql}(s',t), \tag{3.16}$$

where we use $Q_e=-1$. From [47], eq. (3.3.1), we quote:

$$\begin{aligned}\mathscr{A}_Z^{OLA}(s,t) = {} & ie^2\,4I_e^{(3)}I_f^{(3)}\frac{\chi_Z(s)}{s}\rho_{ef}(s,t)\Big\{\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu(1+\gamma_5)\\ & -4|Q_e|s_W^2\kappa_e(s,t)\gamma_\mu\otimes\gamma_\mu(1+\gamma_5)-4|Q_f|s_W^2\kappa_f(s,t)\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu\\ & +16|Q_eQ_f|s_W^4\kappa_{e,f}(s,t)\gamma_\mu\otimes\gamma_\mu\Big\}.\end{aligned} \tag{3.17}$$

The form factors may be used, in analogy to the Z decay matrix element of Sirlin, for definitions of effective vector and axial vector couplings and of an effective weak mixing angle:

$$
\begin{aligned}
G_\mu^{\mathrm{eff}} &= \rho_{ef} G_\mu, && (3.18)\\
\sin^2\theta_{W,e}^{\mathrm{eff}} &= \kappa_e \, \sin^2\theta_W, && (3.19)\\
\sin^2\theta_{W,f}^{\mathrm{eff}} &= \kappa_f \, \sin^2\theta_W, && (3.20)\\
\sin^2\theta_{W,ef}^{\mathrm{eff}} &= \sqrt{\kappa_{ef}} \, \sin^2\theta_W. && (3.21)
\end{aligned}
$$

The unique definition of an effective weak mixing angle is lost.

The first applications of the calculations of weak corrections by the Dubna group were applied, together with N. Shumeiko, to deep-inelastic scattering; see e.g. [48, 49]. The form factors ρ and κ are simply related to the one-loop form factors introduced in the original renormalization articles by Bardin and Fedorenko (1978) [27 – 29] and Bardin, Christova, Fedorenko (1980) [30, 31]:

$$
\begin{aligned}
\rho_{ef} &= 1 + F_{LL}(s,t) - s_W^2 \Delta r, && (3.22)\\
\kappa_e &= 1 + F_{QL}(s,t) - F_{LL}(s,t), && (3.23)\\
\kappa_f &= 1 + F_{LQ}(s,t) - F_{LL}(s,t), && (3.24)\\
\kappa_{ef} &= 1 + F_{QQ}(s,t) - F_{LL}(s,t). && (3.25)
\end{aligned}
$$

The corresponding relations of form factors F_{ij} and Z boson matrix element are:

$$
\begin{aligned}
\mathscr{A}_Z^{OLA} &= i \frac{g^2}{16\pi^2} e^2 4 I_e^{(3)} I_f^{(3)} \frac{\chi_Z(s)}{s} \qquad (3.26)\\
&\times \Big\{ \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu(1+\gamma_5) F_{LL}(s,t) - 4|Q_e| s_W^2 \gamma_\mu \otimes \gamma_\mu(1+\gamma_5) F_{QL}(s,t)\\
&- 4|Q_f| s_W^2 \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu F_{LQ}(s,t) + 16|Q_e Q_f| s_W^4 \gamma_\mu \otimes \gamma_\mu F_{QQ}(s,t) \Big\}.
\end{aligned}
$$

So far we discussed matrix elements. The differential cross section for $e^+e^- \to f\bar{f}$ is:

$$
\begin{aligned}
\frac{d\sigma}{d\cos\vartheta} &= \frac{\pi\alpha_{em}^2}{2s} \Big\{ \left(1+\cos\vartheta^2\right) \left[K_T(\gamma) + \Re e(\chi(s)\ K_T(I)) + |\chi(s)|^2\ K_T(Z)\right] \qquad (3.27)\\
&+ 2\cos\vartheta \left[K_{FB}(\gamma) + \Re e(\chi(s)\ K_{FB}(I)) + |\chi(s)|^2\ K_{FB}(Z)\right] \Big\},
\end{aligned}
$$

with

$$
\chi(s) = \frac{G_F}{\sqrt{2}} \frac{M_Z^2}{8\pi\alpha} \frac{s}{s - M_Z^2 + i\Gamma_Z M_Z}. \qquad (3.28)
$$

One has to care about the choice of a constant or an s-dependent width here [50].

The effective couplings are:

$$\begin{aligned}
K_T(\gamma) =&\ c_{color}\ Q_i^2 Q_f^2 |F_\gamma(s)|^2 && (3.29)\\
=_{Born}&\ c_{color}\ Q_i^2 Q_f^2, &&\\
K_T(I) =&\ 2c_{color}\ |Q_i Q_f|\ F_\gamma(s)^* \rho_{if}(s,t) v_i v_f && (3.30)\\
=_{Born}&\ 2c_{color}\ |Q_i Q_f| v_{B,i} v_{B,f}, &&\\
K_T(Z) =&\ c_{color}\ |\rho_{if}(s,t)|^2 (1+|v_i|^2+|v_f|^2+|v_{if}|^2) && (3.31)\\
=_{Born}&\ c_{color}\ (v_{B,i}^2+a_{B,i}^2)(v_{B,f}^2+a_{B,f}^2), &&\\
K_{FB}(\gamma) =&\ 0, && (3.32)\\
K_{FB}(I) =&\ 2c_{color}\ |Q_i Q_f|\ F_\gamma(s)^* \rho_{if}(s,t) && (3.33)\\
=_{Born}&\ 2c_{color}\ |Q_i Q_f|\ a_{B,i} a_{B,f}, &&\\
K_{FB}(Z) =&\ 2c_{color}\ |\rho_{if}(s,t)|^2 2\Re(v_i v_f + v_{if}) && (3.34)\\
=_{Born}&\ 2c_{color}\ (2v_{B,i} a_{B,i})(2v_{B,f} a_{B,f}). &&
\end{aligned}$$

Here, i denotes the intial state and f the final state. For the Drell-Yan process $\bar{q}q \to l^+l^-$, it is $q = u,d$ and $f = l$.
The c_{color} is the color factor, e.g. $c_{color} = 3$ for initial state quarks and final state leptons.

A similar formula describes the special case of Bhabha scattering [51, 10, 52, 53]. The 1990 article, a numerical comparison with W. Hollik [52], seems to be the most precise prediction for the effective Born cross-section of Bhabha scattering until today.

At the end of the subsection, we would like to emphasize that notions of form factors are not unique. We split, for purely phenomenological reasons, the matrix element into two pieces: a photonic amplitude and a Z boson amplitude. The calculation of the running QED coupling is technically quite different from that of the weak loop diagrams. So this is reasonable. Gauge invariance justifies it, but only if handled with care. There are diagrams which mix a photon and a Z boson amplitude, and this is gauge dependent. So, in ZFITTER we decided to include all the corrections but the fermionic self-energy insertions, a bit arbitrary, into the Z boson amplitude.

Such a separation of photonic and weak terms is wishful also for the charged current W boson mediated amplitude. But a gauge-invariant separation of (virtual and real) photonic corrections from W boson exchange is impossible. In HECTOR [54, 46], we found a way to do well-defined separations by considering logarithmic terms and just explicitely defining some rule. This really worked out. Years later, when building a software for $e^+e^- \to \nu\bar{\nu}\gamma$, we could take over the weak charged current form factor into the Monte Carlo program of S. Jadach and Z. Was [55]. This reaction is, for $\nu = \nu_e$, unique. It depends both on neutral current and charged current amplitudes.[8]

Similar problems have been discussed when the ZFITTER form factors were adapted to atomic violation measurements [56].

3.3 Monte Carlo Programs for Drell-Yan processes at LHC and weak form factors

We went a bit into the details of a correct ansatz for the effective Born approximation in the Standard model at LEP. The situation in a Drell-Yan process is quite similar. One may study e.g. the

---

[8] Bhabha cattering has also s- and t-channel exchanges, but only of neutral current type.

running of the weak mixing angle $\sin^2\theta_W^{\text{eff}}(s')$ as a function of the scale s' from a hard cross-section $\sigma_0(s')$:

$$\sigma_0(s') = \mathscr{L}_u\ \sigma_0(u\bar{u}\to l^+l^-)\ +\ \mathscr{L}_d\ \sigma_0(d\bar{d}\to l^+l^-) \tag{3.35}$$

where both hard scattering cross-sections $\sigma_0(u\bar{u}\to l^+l^-)$ and $\sigma_0(d\bar{d}\to l^+l^-)$ depend on four complex valued, process-dependent form factors $\rho_{ql}, \kappa_q, \kappa_l, \kappa_{ql}$ with $q = u,d$. The σ_0 depends on s', but also on the scattering angle θ. Further, we have not only initial and final state photonic corrections, but also initial-final state interferences.

An elegant way to cover at least part of the complexity of all this in a modern QCD Monte Carlo program is the following:
– Define a photon exchange amplitude.
– Define a Z exchange amplitude.
– Split the v_{ql} into a Z-part and a photon part:

$$v_{ql} \to (v_{ql} - v_q\, v_l)\ +\ v_q\, v_l. \tag{3.36}$$

– Assume a Born like structure with form factors ρ, v_q, v_l and put the deviation from that structure, which is contained in the difference $(v_{ql} - v_q v_l)$, into the photon amplitude.

Evidently, once there are accurate data, one has to carefully understand how to model the correct physics ansatz with a smaller number of parameters. This is under study by experimentalists presently.

4. Real corrections

We started in 1983 to envisage some contribution to the description of the Z boson resonance as it was planned to be studied at LEP. There existed several articles on electroweak radiative corrections. Let us mention the electroweak study by Wetzel in 1982 [57] and that by Lynn and Stuart in 1984 [58], or the MC program MUSTRAAL by Berends, Kleiss, Jadach in 1982 [59]. It was not evident to us that we might contribute some novel results, and we decided therefore to study real photon emission first.

The Dubna group has an enormous experience in the analytical treatment of QED corrections, first mostly applied to t-channel exchange processes. We should mention here the close contacts with Dubna experimentalists of the NA-4 collaboration at CERN. The subtraction method for the treatment of infra-red singularities was worked out in [60]. The divergent part of the cross-section is, in simplified form, integrated over the whole phase space, and at the same time subtracted from the exact squared matrix element. The difference can be integrated numerically, and the isolated term is sufficiently simple for an analytical treatment. In practice, this can become quite involved, see [61].

The first articles treated just photonic corrections, taking into account mass effects. The very first one was on pure QED corrections in e^+e^- annihilation, by A. Akhundov (Baku), D. Bardin (Dubna), O. Fedorenko (Petrozavodsk), T. Riemann (Dubna): “Some Integrals For Exact Calculation Of QED Bremsstrahlung”, an unpublished JINR Dubna preprint [62], followed by [63, 64]. Then we extended the integrational technology to experimental set-ups with Z boson resonance phenomena, including mixing phenomena of Z boson and photon. This sounds easy, but there were several conceptual problems to be solved. Extensive use of SCHOONSCHIP [65, 66] and later of FORM [67, 68] was mandatory. As a result, ZFITTER relies now on several versions of semi-analytical formulae with low-dimensional numerical phase space integrations left. At the time of LEP experiments, this was extremely useful. For an unfolding of measured cross-sections into pseudo-observables, or for multi-dimensional fits, the computing time of an analysis code was absolutely decisive. The inclusion of certain kinematical cuts was a wish expressed by experimentalists. Computers were not so advanced. There were no personal computers, and workstations were also not yet on the market. In Dubna, there were one or two terminal stations for theoreticians, and we had to queue up every day. In Russian Winter, the terminal room (with one terminal) was a bit cold at temperatures close to zero centigrades, because the windows did not close exactly. The upper left corner of the terminal screen was blind. Often the terminal in the theory building was blocked by Riemann, Bardin, Akhundov from 9 to 12 in the morning. Not everybody was amused.

In case of quark-pair production, or Z or W boson decays into quarks, the final state will get QCD modifications. The corrections are contained in so-called radiator functions. Their implementation in ZFITTER relies on calculations by a variety of collegues and is described in the various ZFITTER descriptions, notably in [37, 69, 10, 11]. Useful representations are also e.g. [70–73].

The treatment of the complete set of QED corrections related to real emission of photons in ZFITTER is quite specific. The higher-order corrections have been typically taken over from the literature, as it is documented, notably in [10, 11]. An important example is [74]. The main work had to be performed at one-loop order, plus soft photon exponentiation. It was clear that the numerical effects will be important for the experimental analyses. There were several Monte-Carlo programs available, e.g. [75, 59, 76–79] and the references therein. See also the report [80]. We aimed at an alternative, analytical integration of the three-dimensional photon phase space integrals. The necessary techniques have been developed step by step over a longer period, and originate to a large extent from studies for deep-inelastic scattering, e.g. $lN \to lX$. In the presence of the Z boson resonance in the s-channel, one is faced with the additional need to perform a correct treatment of the Breit-Wigner propagator, a truly complex function. Further, there is a mixing of photon and Z boson exchange. This γZ mixing was studied e.g. in [38, 41, 81, 82]; this issue was settled by a formal Dyson summation of the γ, Z propagator matrix. The Z boson propagator with the finite width may become an issue for analytical integrations. In squared matrix elements, we are faced with $\gamma\gamma$, γZ and ZZ interferences. The latter are dominating around the Z boson pole, and they will contain squared Z boson propagators. To perform analytical phase space integrations with such a term inside looks difficult. An important, simple idea is to perform a partial fraction decomposition in order to linearize the integrand:

$$\begin{aligned}\left|\frac{1}{s-M_Z^2+iM_Z\Gamma_Z}\right|^2 &= \frac{1}{2iM_Z\Gamma_Z}\left(\frac{1}{s-M_Z^2+iM_Z\Gamma_Z}-\frac{1}{s-M_Z^2-iM_Z\Gamma_Z}\right) \\ &= -\frac{1}{M_Z\Gamma_Z}\Re\left(\frac{1}{s-M_Z^2+iM_Z\Gamma_Z}\right).\end{aligned} \tag{4.1}$$

At first glance this looks a bit crazy because the complete answer seems to carry an overall factor $1/(M_Z\Gamma_Z)$. Evidently, one may use complex integration theory, so this is good. The overall prefactor gets divergent for vanishing Z width, but this is the technical expression of the well-know radiative tail, so this is also good.

We tried the approach, and calculated the complete one-loop QED corrections for the total cross-section and the forward-backward asymmetry around the Z resonance without a cut. The results for initial state radiation, final state radiation and the initial final state interferences were rather compact and looked explicitly reasonably behaving.[9] The results were published as preprint in [84] and refined a bit in [85]. The paper could not be published in Nuclear Physics B because the referee found it not close enough to the experimental set-up. Nevertheless, it is a nice piece of work and served for many years as an important numerical etalon for precision comparisons.

As a by-product, we understood that one may calculate the initial-final state interference of the γZ interference as the arithmetical means of the ZZ and $\gamma\gamma$ initial-final state interferences. For a proof see [81]. This is not of utmost importance here. When we later studied QED corrections for Z, Z' production with a heavy Z' boson, then we had at the disposal without a new calculation the newly appearing initial-final state part of the ZZ' interference [86 – 91].

In an unpublished paper [92], we worked out the influence of Z'-physics on the evaluation of weak form factors. This idea of reshuffling matrix elements in form factors was applied (independently) quite recently for a clever inclusion of ZFITTER's weak form factors into LHC Monte Carlo codes, which were originally made for the description of QCD corrections to Drell-Yan processes.[10]

Later we applied the techniques for

– the calculation of integrated cross-sections with soft photon exponentiation [93],

– the corresponding angular distributions [94],

– and the bremsstrahlung integrals with angular cuts to the momenta of one of the final state fermions, [95].

Finally, all this was sufficiently close to what the experimentalists could derive from their Monte-Carlo simulations for a confrontation with theory.

We wrote relatively monstrous programs in Veltman's SCHOONSCHIP [65, 66] to be run at a CDC main frame at JINR Dubna, and later in FORM [67, 68] to be run at personal computers. The first article typeset in latex was presumably [94], and the first one submitted to the hep-ph archive seems also to be [94].

At a certain moment we realized that analytical integrations are fine; but if the sensitivity to the Z boson width becomes sufficiently large, then it will matter whether the width is a pure constant γ_Z as in a normal Breit-Wigner function, or whether it arises from a quantum field theoretical calculation and will thus depend on the kinematics, $\Gamma_Z(s)$. In the latter case, it is (roughly speaking) the imaginary part of the Z boson self-energy function, which is by itself s-dependent; and for initial state corrections s'-dependent. The s' is one of the integration variables. We remembered that the s-dependence is to a very high accuracy is just $M_Z\,\Gamma_Z(s) \sim (s/M_Z)\,\Gamma_Z$, and this observation enables us to change the propagators into functions with a constant width, allowing not only a good estimate

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[9] In fact, it took us nearly half a year of heavy fighting with SCHOONSCHIP, because we did not agree, at the Z boson peak, with the numerics of the Monte-Carlo program MUSTRAAL [59, 83]. The MUSTRAAL was available via CPC, and we could run it at Dubna. The mistake was, as often, trivial, but influential. The final 5 digits agreement convinced us that our Breit-Wigner treatment makes sense and is operational.

[10] W. Sakumoto, private information.

of the different approaches, but also furtheron the analytical integrations. The differences of mass and width in the two approaches are [50]:[11]

$$m_Z = M_Z - \frac{\Gamma_Z^2}{2M_Z} = M_Z - 34 \text{ GeV}, \tag{4.2}$$

$$\gamma_Z = \Gamma_Z - \frac{\Gamma_Z^3}{2M_Z^2} = \Gamma_Z - 0.934 \text{ MeV} \approx -1 \text{ MeV}. \tag{4.3}$$

Here, $M_Z = 91187.6$ GeV and $\Gamma_Z = 2.4952$ GeV have to be chosen as the usual PDG values. Later we worked out an approach to a model-independent Z boson peak analysis inspired by S-matrix theory. Not only for the Z boson peak cross-section, but also for asymmetries. The point here again is a proper treatment of QED corrections [97 – 102].[12]

In fact, the idea was born during a talk on string theory at a conference, while reading a paper on QED corrections with complicated phase space cuts by Passarino [103]. The sophisticated final state phase space treatment of [103] was relatively close to experimental cuts for lepton final states, using as a cut variable the acollinearity of the leptons. Analytical QED corrections were worked out for this case in [104] and became part of ZFITTER. The truly nice paper remained unpublished, unfortunately. Later, we recalculated these corrections for ZFITTER from the scratch (unpublished, see [105 – 108]) and got very nice, compact formulae for the special case of no cut for the fermion production angle [109].

The Z boson parameter relations 4.2 and 4.3 (we did not mention here modifications of the pole residue) becomes essential when two-loop electroweak corrections are determined in ZFITTER. This is carefully described in [110], where the complete electroweak two-loop corrections to the leptonic weak mixing angle have been calculated. See also section 6. It is remarkable that the shift of the Z boson width due to the change of scheme (s-dependence or constant width) amounts to 1 MeV and is larger than the corresponding shift from the genuine weak NNLO corrections. Compared to the experimental error of 2.3 MeV, it is small. The authors of [110] did not take the correction into account because it is formally beyond the NNLO order and thus among the systematically neglected terms.[13] One should consider the term as an indication of the size of unknown higher order terms.

What we described here was about the state of real emission affairs in ZFITTER at the end of the nineteen-eighties. Final state mass effects treatments were refined in [111 – 113]. Some additional QED corrections, due to light fermion pair emission and higher order photonic effects, needed for a proper treatment at LEP 2 energies were later added [114, 115]. See also [116].

Careful studies of ZFITTER physics updates originated in these years [117, 117, 72, 118].

5. Competition and cooperation

5.1 1989 - First LEP publications

In 1989, the world changed quite a bit. Participation at the Ringberg Workshop on LEP physics in Germany became possible [119]. The NATO supported RADCOR conference on radiative cor-

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[11] The Z boson mass shift was discovered by a numerical study of the Z boson peak in parallel in [96].

[12] The software package SMATASY is supported by Martin Grünewald.

[13] Ayres Freitas, private information.

rections and their applications to experiments in Brighton, the first one of a series, was open to Eastern Country physicists [120]. We remember the stimulating athmosphere of the 1989 LEP physics workshop at CERN [121, 122]. And LEP became operative in July 1989. The first months were exciting. A good knowledge of radiative corrections was needed from the very beginning, just in order to discriminate between trivial radiative effects and New Physics. Several unplublished ZFITTER related theory studies appeared in this period, e.g. [123 – 126]. The LEP collaborations performed the first line shape analyses. We were closely related to L3 [127 – 129] and DELPHI [130 – 134].

Among the first DELPHI papers was [134]. From the ZFITTER group, D. Bardin and G. Mitselmaker were DELPHI authors. The paper quotes for the theory on the Z line shape G. Burgers [135] and A. Borrelli et al. [136]. In [132], the Z line shape analysis used the software packages ZAPPH and ZHADRO by G. Burgers [135]. In [131], March 1990, our papers [37, 104] are quoted. And in [133] the package ZFITTER/ZBIZON with reference to the internal note DELPHI 89-71 PHYS 52 and to [37, 9] was used.[14]

A similar approach was observed in the L3 collaboration, were ZFITTER authors T. Riemann, M. Sachwitz and H. Vogt were collaborating. The internal note L3-001 [127] quotes G. Burgers [135] and CERN 89-08, but also our paper [50]. The Z line shape analysis seems to be based on papers by Cahn [137] and Borrelli et al. [136]. In [129], internal note L3-003, our package ZBIZON is quoted with reference to L3 Internal Note 679 as well as [50] and the Zeuthen preprint PHE 89-19 [94]. Back-up radiative corrections had been studied with ZBIZON. For the very Z line shape fits they used again Borrelli et al., Cahn, and a paper by Jadach et al. (for Bhabha scattering). In [128], internal note L3-004, the paper on the Z boson parameter [50] was quoted.

A bit later it became more and more common to use ZFITTER in DELPHI and L3, but also in OPAL. While ALEPH used the package BHM/WOH by F. Berends, M. Martinez, W. Hollik et al. [138, 70]. We mention these very first papers on LEP physics results because they demonstrate that there was a true competition of the analysis packages and our ZBIZON/ZFITTER package was accepted step by step, but not from the very beginning.

5.2 1992-2012 - LEPEWWG and global fits

The LEP Electroweak Working Group was founded in 1993.[15] Soon after the first measurements at LEP the quest was expressed for combined data analyses with a fourfold statistics compared to a single experiment. Originally a group with members of the four LEP experiments, led by Jack Steinberger, investigated the combination of the Z line shape [139]. In 1993 Dorothee Schaile was asked to take over the coordination of the group and she had then already ideas on the inclusion of other electroweak observables into a combined analysis. They called themselves the LEP EWWG. The first publicly accessible document with this name is also the initial summary of the LEP results for the electroweak Summer conferences in 1993, which then appeared annually [140 – 142]. The LEP EWWG was lead by D. Schaile from 1993-1996. When she became professor in Munich, Martin Gruenewald took over the coordination of the LEP EEWG. The final

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[14] ZBIZON is the former version of ZFITTER.

[15] We thank Dorothee Schaile for private information, January 2012.

paper on LEP 1 data appeared in 2005 [23], nearly a decade after closing LEP 1 in 1996, while the analysis of LEP2 data (finalized data taking in 2000) is presently finalizing.

The ZFITTER group members, as well as the authors of other physics software packages used by the LEPEWWG are not members of the LEPEWWG. They are consulted in case.

5.3 1995 – The Electroweak Working Group Report

The work of the LEPEWWG and of the four LEP collaborations relied on ZFITTER and TOPAZ0, and also on the BHM/WOH package, and on many other resources. Because of this role of establishing a kind of world standard, the community felt the need of careful numerical checks on their predictions. One is confronted with multi-parameter problems, different calculational schemes, some freedom of input choices, in the presence of approximations and dedicated omissions, of misunderstandings and, sometimes, mistakes.

At a certain moment, the community has to set benchmarks. The result of a year-long workshop is the collection "Reports of the working group on precision calculations for the Z resonance" , edited by D. Bardin, W. Hollik, G. Passarino. It was published as Yellow CERN Report, CERN 95-03 (31 March 1995), `http://cdsweb.cern.ch/record/280836/files/CERN-95-03.pdf`.

Part of this document is the "Electroweak Working Group Report", which was two years later submitted to the archive arxive/hep-ph [70].[16] This work is one of the basics for the successful work of the LEP Electroweak Working Group. It is until now one of the most important collections of Standard Model higher order corrections for e^+e^--annihilation.

5.4 Higher order corrections in ZFITTER

During the 1995 CERN workshop and shortly after, a lot of additional higher-order corrections were calculated and included into ZFITTER. We give here just a (presumably not complete) list of the references and refer for any detail to the ZFITTER descriptions: [71, 82, 110, 143 – 150]. Later, further improvements were added [151 – 160].

Until now, we did not yet include into ZFITTER the existing parameterization of the rather small *bosonic* two-loop weak corrections to the weak mixing angle [158]. The fermionic corrections are covered, as well as the complete weak two-loop corrections to the W boson mass. For a complete treatment of the weak two-loop corrections to the Z boson width, the corrections to the form factor ρ_Z are lacking yet. For this reason, the quite good agreement of the higher-order *approximations* to Γ_Z with the so far known pieces of the *complete* two-loop result are an indication that the final answer will be close what we have already.

Generally speaking, we try to control about four to five digits of the predictions aiming at such a *physical* theory precision. One quote from the report [70] is interesting because it sheds some light on the progress of the so-called *technical* precision (precision under fixed, maybe not realistic conditions): “ ... compare results of independent calculations. Such a comparison has been done once for Δr, and an agreement of up to 12 digits (computer precision) was found [14].” Ref. [14] was private communications of Bardin, Kniehl, Stuart, 1992. This has to be compared to a three digits agreement between two Bhabha cross section calculations in a comparison, performed few

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[16] Now it is also available as a pdf file at CERN, in CERN 95-03.

years earlier in 1990 [52]. Later, in 2002, a precision of up to 12 digits was reached in practice for complete virtual one-loop calculations, and of 5 digits with inclusion of real corrections [161 – 163].

6. ZFITTER 2013

6.1 From ZFITTER v.6.42 to ZFITTER v.6.44beta

The most recent publicly available ZFITTER version is ZFITTER v.6.43 (17 June 2008) [10, 11]. It agrees with ZFITTER v.6.42 up to a correction of a non-influential typo and was released by the ZFITTER support group (A. Arbuzov, M. Awramik, M. Czakon, A. Freitas, M. Grünewald, K. Mönig, S. Riemann, T. Riemann, see `http://zfitter.com`). The ZFITTER group was reorganized in February 2012 and consists now of A. Akhundov, A. Arbuzov, D. Bardin, P. Christova, L. Kalinovskaya, A. Olshevksy, S. Riemann, T. Riemann.

Recently, we have included into ZFITTER v.6.44beta the final results for the $\mathscr{O}(\alpha_s^4)$ QCD corrections to the Z-boson and W-boson quarkonic partial widths and to the so-called R-ratio by P. Baikov et al. [164, 160]. As may be seen from figure 6.1 and from table 6.2, the numerical shifts in the widths amount to less than 0.3 MeV and are thus well below the experimental errors, e.g. at LEP or at an anticipated GigaZ option of an ILC [165]. A fit formula for the complete electroweak two-loop corrections to the W-boson mass [154] was already included in ZFITTER v.6.42. The final exact results for the complete electroweak two-loop corrections to $\sin^2\theta_{\mathrm{eff}}^{\mathrm{f\bar{f}}}$ for light fermions f [110] and the two-loop electroweak fermionic corrections to $\sin^2\theta_{\mathrm{eff}}^{\mathrm{b\bar{b}}}$ [159] have to be included yet into ZFITTER. They are known to be small corrections compared to the fit formula [157] covered in ZFITTER since v.6.42. Already these corrections are small compared to the present experimental errors, see table 6.1.

Presently, there are controversial positions concerning ZFITTER's 'conditions of use' and the ZFITTER software licence `http://cpc.cs.qub.ac.uk/licence/licence.html` granted to the authors by Elsevier's Computer Physics Communications Program Library - Programs in Physics & Physical Chemistry. For some details see `http://zfitter.com`. Until the issue is settled, actualized versions of ZFITTER will stay at the beta level and cannot be released.

Sooner or later, the LHC is becoming a precision tool and the community feels some steady need of high-precision Standard Model predictions. Both for use in global fits and for specific cross-section predictions, notably of Drell-Yan processes via the Z resonance. This need would become even more pronounced if the ILC project would substantialize [165].

Regrettably, we see today no alternative project to ZFITTER in the field of precision Standard Model predictions. In the mid-nineteen nineties there were three competing (and cooperating) projects at the disposal [70]: BHM/WOH by W. Hollik et al., TOPAZ0 by G. Passarino et al., and ZFITTER by D. Bardin et al. BHM/WOH was available on request, and the latter two are publicly available. To our knowledge, updating and user support have been minimized for TOPAZ0 and BHM/WOH [138].

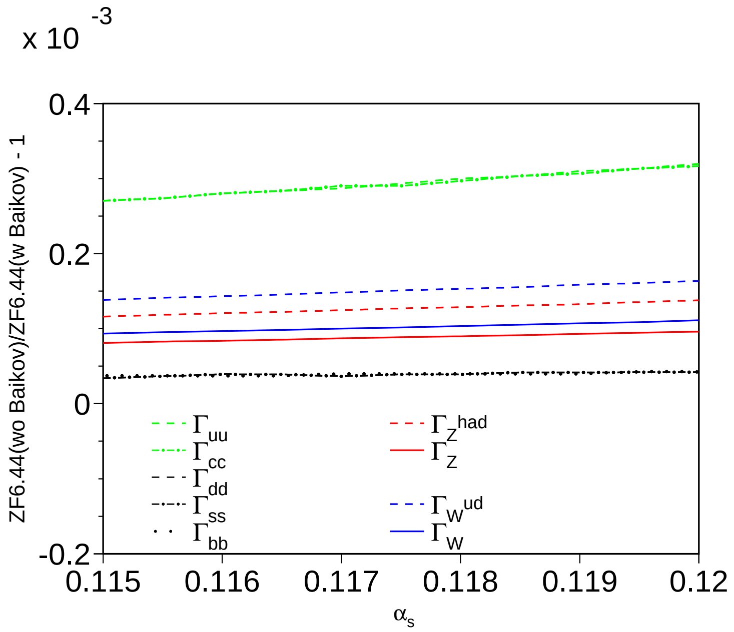

Figure 6.1: *The influence of the* $\mathcal{O}(\alpha_s^4)$ *QCD corrections [160] on the W and Z boson widths.*

Table 6.1: *ZF6_44, with the input values* α_s *= 0.1184(7),* M_Z *= 91.1876(0),* M_W *= 80.385(15),* M_H *= 125(0),* m_t *= 173.5(1). The dependence on electroweak NNLO corrections is studied for IMOMS=1 (input values are* α_{em}, M_Z, G_μ*). IAMT4= 4: with two-loop sub-leading corrections and re-summation recipe of [23-28] of [11]; IAMT4=5: with fermionic two-loop corrections to* M_W *according to [29,30,32] of [11]; IAMT4=6: with complete two-loop corrections to* M_W *[37] and fermionic two-loop corrections to* $\sin^2\theta_W^{\text{lept,eff}}$ *[52] of [11].*

IAMT4	4	5	6	diff.	exp. error
$\Gamma_Z(\mu\mu)$	83.978	83.975	83.981	0.006	0.086
Γ_Z	2494.8	2494.6	2494.9	0.3	2.3
$\Gamma_W(l\nu)$	226.32	226.29	226.29	0.03	1.9
Γ_W	2090.3	2090.0	2090.1	0.2	42
M_W	80.358	80.354	80.355	0.004	0.015

6.2 Gfitter

Sometimes the Gfitter project is considered as an independent implementation of Standard Model predictions for some pseudo-observables, and as a true scientific alternative to ZFITTER (for these pseudo-observables). We do not share this opinion and would like to give a short, clarifying comment on the situation. There are several versions of the program Gfitter.

The Gfitter project was started in 2006 and presented to the public in December 2007, at the kick-off meeting of the German “Helmholtz Alliance for Physics at the Terascale”, see the slides at `http://indico.desy.de/materialDisplay.py?contribId=36&sessionId=15&materialId=1&confId=477`. Until August 2012, the

Table 6.2: *IBAIKOV=0 (no α_s^4 QCD corrections) or IBAIKOV=2012 [160], IAMT4 as described in table 6.1.*

IBAIKOV=0 IAMT4	4	5	6	Diff.	Exp. Err
$\Gamma_Z(\mu\mu)$	83.9782	83.9748	83.9807		0.086
Γ_Z	2494.7863	2494.0465	2494.8688		2.3
$\Gamma_W(l\nu)$	226.3185	226.2877	226.2922		1.9
Γ_W	2090.3308	2090.0465	2090.0882		42
M_W	80.3578	80.3541	80.3546		0.015
s2efflept	0.231722	0.231791	0.231670		
IBAIKOV=2012 IAMT4	4	5	6	Diff.	Exp. Err
$\Gamma_Z(\mu\mu)$	83.9782	83.9748	83.9807		0.086
Γ_Z	2494.5591	2494.3747	2494.6416		2.3
$\Gamma_W(l\nu)$	226.3185	226.2877	226.2922		1.9
Γ_W	2090.1117	2089.8274	2089.8691		42
M_W	80.3578	80.3541	80.3546		0.015
s2efflept	0.231722	0.231791	0.231670		

Gfitter software was proprietary, but by private information[17] it became known that the Standard Model library of Gfitter, Gfitter/GSM, was relying on the FORTRAN package ZFITTER v.6.42 and was created to a large extent by copy-paste-adapt. Without any proper citation in the academic meaning of the word.

Gfitter/GSM (Summer 2006 - July 2011) relies essentially and directly on the Standard Model implementation of the ZFITTER software. On top of that, Gfitter/GSM contains few add-ons. The *electroweak add-on* of Gfitter/GSM, compared to ZFITTER v.6.42, are the bosonic two-loop corrections to the weak mixing angle in Amwramik et al. [110]. They are small; see the discussion above. The complete two-loop parameterizations in [110], in turn, have been made with use of ZFITTER v.6.42. As a consequence, it is formally correct to quote for the parameterization only [110], but one should have in mind that there is inside also ZFITTER. There is also a *QCD add-on* of Gfitter/GSM (2011), compared to ZFITTER v.6.42 (2006), based on [166]. It is also numerically small (see the discussion above) and is implemented in ZFITTER v.6.44beta.

Gfitter/GSM (August 2011 till August 2012) relied on a proprietary implementation of Standard Model corrections which were based on a parameterization tracing back to Cho et al. (1999) [167], which in turn is based on an electroweak one-loop calculation published in 1994 [168]. There have been made improvements later, and in a recent article by Cho et al. (2011) [169] the authors confirm the reliability of their parametrization by comparing them with ZFITTER v.6.42 predictions. These parameterizations are used in Gfitter furtheron, and overlayed with the most recent higher-order corrections mentioned.

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[17] Private information from and documentation by A. Akhundov, S. Riemann, T. Riemann, March to May 2011.

Gfitter_1.0 has been released publicly in September 2012. The Standard Model library Gfitter_1.0/gew relies presumably on the same parameterizations as Gfitter/GSM (2012).

The different versions of Gfitter rely in one way or the other on ZFITTER v.6.42. We further remark that without studying the numerical reliability of Gfitter, to four or five significant digits, the scientific value of the inclusion of NNLO weak and α_s^4 QCD corrections in Gfitter remains questionable. According to our standards, Gfitter simulates Standard Model predictions with unknown precision. It is a nice tool for the production of figures for the illustration of Standard Model physics. Possibly it is useful for studies beyond the Standard Model.

7. Conclusions

This talk was presented at LL2012, the eleventh "Loops and Legs" meeting. This conference was founded by the Zeuthen Theory Group in 1992 when the Zeuthen Institute for High Energy Physics of the (then already former) East German Academy of Sciences became part of DESY. We are glad that this conference attracts since then regularly collegues who contribute to the progress in the field. A field, comprising both the branch of applied calculations and that of development of new theoretical methods.

ZFITTER is certainly one of the oldest source-open software projects in elementary particle physics with a permanent support. It comprises practically all the theoretical knowledge of relevance for a precise description of the Z boson resonance in e^+e^- annihilation and for Z boson's part in global fits in the Standard Model [170]. Certainly, today one would create such a project quite differently. We can only encourage our collegues to try. Complex projects need (independent) duplication.

Higher order quantum field theoretical predictions face another problem: The solutions become so lengthy and complex that the idea of source-open software is, in practice, no longer a realistic option. This happens already with the $\mathscr{O}(\alpha_s^4)$ QCD corrections and the complete NNLO weak corrections in ZFITTER. They are mere parameterizations of huge, unpublished expressions.

The LEP/SLC era gave the scientific community unprecedented precision in several fundamental quantities like M_Z, Γ_Z, the effective weak mixing angle $\sin^2\theta_W^{\rm eff}$, the number of light neutrino flavors N_ν. Of comparable importance is the experimental confirmation of the Standard Model, a gauge theory with spontaneous symmetry breaking, as a consistent quantum filed theory, with inclusion of higher orders of perturbation theory.

We are proud that we are being contributing.

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